MOTION IN A PLANE LEVEL 2
SET 1
SOLUTION

1 **(d)**

Given, $x = 0.20$ m, $y = 0.20$ m, $u = 1.8\text{ms}^{-1}$

Let the ball strike the nth step of stairs,

Vertical distance travelled

$= ny = n \times 0.20 = \frac{1}{2}gt^2$

Horizontal distance travelled, $nx = ut$

or $t = \frac{nx}{u}$

$\therefore ny = \frac{1}{2}g \times \frac{n^2x^2}{u^2}$

or $n = \frac{2u^2}{g}\frac{y}{x^2} = \frac{2 \times (1.8)^2 \times 0.20}{9.8 \times (0.20)^2}$

$= 3.3 = 4$

2 **(a)**

$\frac{T_{\max}}{T_{\min}} = \frac{\frac{mv^2}{L}+mg}{\frac{mv^2}{L}-mg} = 2 \quad$ ...(i)

Simplifying Eq. (i), we get,

$v_H = \sqrt{3gL} = \sqrt{\frac{3 \times 10 \times 10}{3}} = 10\text{ ms}^{-1}$

3 **(b)**

The two angles of projection are clearly θ and $(90° - \theta)$

$T_1 = \frac{2v\sin\theta}{g}$ and $T_2 = \frac{2v\sin(90° - \theta)}{g}$

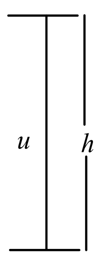
$T_1T_2 = \frac{2(v)^2(2\sin\theta\cos\theta)}{g \times g} = \frac{2R}{g}$

4 **(c)**

$h = \frac{u^2}{2g} \Longrightarrow u^2 = 2gh$

Maximum horizontal distance

$R_{\max} = \frac{u^2}{g}$



$R_{\max} = 2h$

5 **(c)**

$R = 4H\cot\theta$

When $R = H$ then $\cot\theta = 1/4 \Rightarrow \theta = \tan^{-1}(4)$

6 **(c)**

$$h_1 = \frac{v^2 \sin^2 \alpha}{2g}, h_2 = \frac{v^2 \cos^2 \alpha}{2g}, \frac{h_1}{h_2} = \tan^2 \alpha$$

7 **(c)**

$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

$$\mathbf{L} = m\left[v_0 \cos\theta\, t\hat{\mathbf{i}} + \left(v_0 \sin\theta\, t - \frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right]$$

$$\times \left[v_0 \cos\theta\, \hat{\mathbf{i}} + (v_0 \sin\theta - gt)\hat{\mathbf{j}}\right]$$

$$= mv_0 \cos\theta\, t\left[-\frac{1}{2}gt\right]\hat{\mathbf{k}}$$

$$= -\frac{1}{2}mgv_0 t^2 \cos\theta\, \hat{\mathbf{k}}$$

8 **(b)**

$v^2 = u^2 + 2as$

At max. height $v = 0$ and for upward direction $a = -g$

$$\therefore u^2 = 2gs \Rightarrow s = \frac{u^2}{2g};\ \because s_e = s_p$$

$$\left(\frac{u_e}{u_p}\right)^2 = \left(\frac{g_e}{g_p}\right) \Rightarrow \left(\frac{5}{3}\right)^2 = \frac{9.8}{g_p} \Rightarrow g_p = 3.5\, m/s^2$$

9 **(b)**

Reaction on inner wheel $R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right]$

Reaction on outer wheel $R_2 = M\left[g + \frac{v^2 h}{ra}\right]$

where, r = radius of circular path, $2a$ =distance between two wheels and h = height of centre of gravity of car

10 **(d)**

$$\frac{mv^2}{r} = 10 \Rightarrow \frac{1}{2}\, mv^2 = 10 \times \frac{r}{2} = 1\, J$$

11 **(b)**

$$H = \frac{v^2 \cos^2 \beta}{2g} \text{ or } v \cos \beta = \sqrt{2gH}$$

$$t = \frac{v \cos \beta}{g} = \frac{\sqrt{2gH}}{g} \text{ or } t = \sqrt{\frac{2H}{g}}$$

12 **(c)**

$\tan 90° = \frac{B \sin\theta}{A + B\cos\theta}$ or $A + B\cos\theta = 0$

or $\cos\theta = -A/B$ ....(i)

$R = \frac{B}{2} = [A^2 + B^2 + 2AB\cos\theta]^{1/2}$

or $\frac{B^2}{4} = A^2 + B^2 + 2AB(-A/B) = B^2 - A^2$

or $\frac{A^2}{B^2} = \frac{3}{4}$ or $\frac{A}{B} = \frac{\sqrt{3}}{2}$

From (i), $\cos\theta = -\frac{\sqrt{3}}{2} = \cos 150°$

13 **(b)**

$\vec{P} + \vec{Q} = \hat{i}$

$\vec{Q} = \hat{i} - \hat{i} + \hat{j} - \hat{k}$

$= \hat{j} - \hat{k}$

14 **(b)**

Let the angle of projection be α.

$\therefore$ Range, $R = \dfrac{u^2 \sin 2\alpha}{g}$

and maximum height $H = \dfrac{u^2 \sin^2\alpha}{2g}$

Now, it is given that,

$(\text{Range})^2 = 48(\text{maximum height})^2$

$\therefore \left(\dfrac{u^2 \sin 2\alpha}{g}\right)^2 = 48\left(\dfrac{u^2 \sin^2\alpha}{2g}\right)^2$

or $\dfrac{u^2 \sin 2\alpha}{g} = 4\sqrt{3}\left(\dfrac{u^2 \sin^2\alpha}{2g}\right)$

or $\dfrac{2\sin\alpha\cos\alpha}{4\sqrt{3}} = \dfrac{\sin^2\alpha}{2}$

or $\tan\alpha = \dfrac{4}{4\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore\ \alpha = 30°$

15 **(d)**

$\vec{A} \times \vec{B} = (\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})$

$= -2\hat{k} - \hat{j} - 6(-\hat{k}) - 2\hat{i} + 9\hat{j} - 6(-\hat{i}) = 4\hat{i} + 8\hat{j} + 4\hat{k}$

Modulus is $\sqrt{4^2 + 8^2 + 4^2} = \sqrt{32 + 64}$

$= \sqrt{96} = 4\sqrt{6}$ units.

16 **(c)**

From $v = r\omega$, when v is doubled and ω halved, r must be 4 times. Therefore, centripetal acceleration $= \frac{v^2}{r} = r\omega^2$ will remain unchanged

17

18 **(c)**

$R_{\max} = \dfrac{u^2}{g} = 100 \Rightarrow u = 10\sqrt{10} = 32\ m/s$

19 **(a)**

In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence angular momentum about centre remains conserved.

20 **(a)**

$\vec{F} = F_x\hat{i} + F_y\hat{j}$ or $\vec{F} = 2\hat{i} - 3\hat{j}$.

21 **(d)**

$h = \dfrac{5}{2}r \Rightarrow r = \dfrac{2}{5} \times h = \dfrac{2}{5} \times 5 = 2\ metre$

22 **(d)**

$p = mv\cos\theta$

$= 1 \times 10 \times \cos 60° = 10\left(\dfrac{1}{2}\right)\text{kg ms}^{-1} = 5\ \text{kg ms}^{-1}$

23 **(d)**

$v\cos\beta = u\cos\alpha$

$v = \dfrac{u\cos\alpha}{\cos\beta}$

24 **(d)**

Here, $r = 50$ m

As $\tan\theta = \frac{v^2}{rg}$, therefore, when speed v is doubled; r must be made 4 times, if θ remains the same

∴ New radius of curvature,

$r' = 4r = 4 \times 50 \text{ m} = 200 \text{ m}$

25 **(c)**

Frequency of wheel, $v = \frac{300}{60} = 5$ rps. Angle described by wheel in one rotation $= 2\pi$ rad. Therefore, angle described by wheel in 1 s $= 2\pi \times 5$ rad $= 10\pi$ rad

26 **(b)**

Here, $m = 5$ kg, $r = 2$m, $v = 6 \text{ ms}^{-1}$

The tension is maximum at the lowest point

$$T_{\max} = mg + \frac{mv^2}{r}$$

$$= 5 \times 9.8 + \frac{5 \times 6 \times 6}{2}$$

$$= 139\text{N}$$

27 **(c)**

$\alpha = \frac{\omega}{t}$ and $\omega = \frac{\theta}{t}$

$$\therefore \alpha = \frac{\theta}{t^2}$$

But α =constant

So, $\frac{\theta_1}{\theta_1+\theta_2} = \frac{(2)^2}{(2+2)^2}$

or $\frac{\theta_1}{\theta_1+\theta_2} = \frac{1}{4}$

or $\frac{\theta_1+\theta_2}{\theta_1} = \frac{4}{1}$

or $1 + \frac{\theta_2}{\theta_1} = \frac{4}{1}$

$$\therefore \frac{\theta_2}{\theta_1} = 3$$

28 **(d)**

$\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$; $A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\cos\theta = \frac{\vec{A}.\hat{\imath}}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$$= \frac{1.732}{3} = 0.5773 = \cos 54°44'$$

$\theta = 54°44'$

29 **(d)**

For body to move on circular path. Frictional force provides the necessary centripetal force,

ie, frictional force = centripetal force

or $\mu mg = \frac{mv_0^2}{r} = mr\omega^2$

or $\mu g = r\omega^2$

$\therefore 0.5 \times 9.8 = 10\,\omega^2$

or $\omega = 0.7 \text{ rad s}^{-1}$

30 **(d)**

Horizontal range, $R = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{g}$

Maximum height, $H = \frac{u^2 \sin^2 45°}{g} = \frac{u^2}{4g}$

$\therefore \frac{R}{H} = \frac{4}{1}$

31 **(b)**

Given $(KE)_{\text{highest}} = \frac{1}{2}(\text{KE})$

$\frac{1}{2}mv^2 \cos^2 \theta = \frac{1}{2} \cdot \frac{1}{2}mv^2$

$\cos^2 \theta = \frac{1}{2} \Rightarrow \cos \theta = \sqrt{\frac{1}{2}}$

$\Rightarrow \theta = 45°$

32 **(a)**

The horizontal range $R_x = \frac{u^2 \sin 2\theta}{g}$

When projected at angle of 15°

$R_{x1} = \frac{u^2}{g} \sin(2 \times 15) = \frac{u^2}{2g} = 1.5\text{km}$

When projected at angle of 45°

$R_{x1} = \frac{u^2}{g} \sin(2 \times 45°) \frac{u^2}{g}$

$= \frac{2u^2}{2g} = 2 \times 1.5 = 3.0 \text{ km}$

33 **(b)**

For banking $\tan \theta = \frac{V^2}{Rg}$

$\tan 45 = \frac{V^2}{90 \times 10} = 1$

$V = 30\ m/s$

34 **(a)**

$\vec{A} = A\vec{A}$ or $\vec{A} = \frac{\vec{A}}{A}$

$\therefore$ required unit vector is $\frac{\hat{i}+\hat{j}}{|\hat{i}+\hat{j}|} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

35 **(a)**

$a = \omega^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5\ m/s^2$

36 **(b)**

$R_{\max} = \frac{u^2}{g} = 16 \times 10^3$

$\Rightarrow u = 400 m/s$

37 **(c)**

As, $\vec{A} \cdot \vec{B} = 0$ so $\vec{A}$ is perpendicular to $\vec{B}$. Also $\vec{A}.\vec{C} = 0$ means $\vec{A}$ is perpendicular to $\vec{C}$. Since $\vec{B} \times \vec{C}$ is perpendicular to $\vec{B}$ and $\vec{C}$, so $\vec{A}$ parallel to $\vec{B} \times \vec{C}$.

38 **(c)**

They have same ω

Centripetal acceleration $= \omega^2 r$

$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

39 **(d)**

Let $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ be the initial velocities of the two particles and θ_1 and θ_2 be their angles of projection with the horizontal

The velocities of the two particles after time t are,

$\vec{\mathbf{v}}_1 = (u_1 \cos\theta_1)\hat{\mathbf{i}} + (u_1 \sin\theta_1 - gt)\hat{\mathbf{j}}$ and

$\vec{\mathbf{v}}_1 = (u_1 \cos\theta_1)\hat{\mathbf{i}} + (u_2 \sin\theta_2 - gt)\hat{\mathbf{j}}$

Their relative velocity is $\vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2$

$= (u_1 \cos\theta_1 - u_2 \cos\theta_1)\,\hat{\mathbf{i}} + (u_1 \sin\theta_1 - u_2 \sin\theta_2)\hat{\mathbf{j}}$

Which is a constant. So the path followed by one, as seen by the other is a straight line, making a constant angle with the horizontal

40 **(c)**

$P + Q = 16$ (i)

$P^2 + Q^2 + 2PQ\cos\theta = 64$ (ii)

$\tan 90° = \frac{Q\sin\theta}{P+Q\cos\theta}$

$$\infty = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$\Rightarrow \quad P + Q\cos\theta = 0$ or $Q\cos\theta = -P$

From Eq. (ii)

$P^2 + Q^2 + 2P(-P) = 64$ or $Q^2 - P^2 = 64$

or $(Q - P)(Q + P) = 64$

or $Q - P = \frac{64}{16} = 4$ (iii)

Adding Eq. (i) and (iii), we get

$2Q = 20$ or $Q = 10$ units

From (i), $P + 10 = 16$ or $P = 6$ units

41 **(c)**

Let A and B be the two forces. As per question

$\sqrt{A^2 + B^2} = 5$

or $A^2 + B^2 = 25$ (i)

and $A^2 + B^2 + 2AB\cos 120° = 13$

or $25 + 2AB \times (-1/2) = 13$

or $AB = 25 - 13 = 12$

or $2AB = 24$ (ii)

Solving (i) and (ii), we get

$A = 3$N

and $B = 4$ N

42 **(a)**

$$\frac{a_R}{a_r} = \frac{\omega_{R\times R}^2}{\omega_r^2 \times r} = \frac{T_r^2}{T_R^2} \times \frac{R}{r} = \frac{R}{r} [As\ T_r = T_R]$$

43 **(c)**

$x = 20 \times 5 = 100$m

$$y = \frac{1}{2} \times 10 \times 5 \times 5 = 125\text{m}$$

$r = \sqrt{100^2 + 125^2} = 160$m

44 **(a)**

Initial angular velocity $\omega_0 = 0$. Final angular velocity $\omega = \frac{v}{r} = \frac{80}{(20/\pi)} = 4\pi$ rad s^{-1}

angle described, $\theta = 4\pi$ rad

$\therefore$ Angular acceleration, $\alpha = \frac{\omega^2-\omega_0^2}{2\theta}$

$= \frac{(4\pi)^2-0}{2\times4\pi} = 2\pi$ rad s^{-2}

Linear acceleration, $a = \alpha r$

$= 2\pi \times \frac{20}{\pi} = 40$ ms^{-2}

45 **(b)**

Maximum height $H = \frac{v^2\cos^2\beta}{2g}$

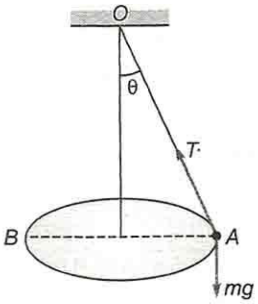
or $v\cos\beta = \sqrt{2gH}$

$t = \frac{v\cos\beta}{g} = \frac{\sqrt{2gH}}{g}$

$t = \sqrt{\frac{2H}{g}}$

46 **(d)**

In figure, $\sin 30° = \frac{AB}{OA}$



or $OA = \frac{AB}{\sin 30°} = \frac{4}{1/2} = 8$m

$\frac{T}{AO} = \frac{F}{AB} = \frac{mg}{OB}$

$F = \frac{AO}{AB} \times F = \frac{AO}{AB}\frac{mv^2}{r} = \frac{8}{4} \times 10 \times \frac{5^2}{4} \approx 125$ N

47 **(c)**

The body crosses the top most position of a vertical circle with critical velocity, so the velocity at the lowest point of vertical circle $u = \sqrt{5gr}$

Velocity of the body when string is horizontal is

$v^2 = u^2 - 2gr = 5gr - 2gr = 3gr$

$\therefore$ Centripetal acceleration $= \frac{v^2}{r} = \frac{3gr}{r} = 3$g

48 **(a)**

Let $\vec{A} + \vec{B} = \vec{R}$. Given A_x=7 and $A_y = 6$

Also $R_x = 11$ and R_y=9. Therefore,

$B_x = R_x - A_x = 11 - 7 = 4$

and $B_y = R_y - A_y = 9 - 6 = 3$

Hence, $B = \sqrt{B_x^2 + B_y^2} = \sqrt{4^2 + 3^2} = 5$

49 **(b)**

Net acceleration in nonuniform circular motion,

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7\ m/s^2$$

a_t = tangential acceleration

a_c = centripetal acceleration $= \frac{v^2}{r}$

50 **(c)**

Here, $r = 300$ m, $\mu = 0.3$, $\mathrm{g} = 10$ ms^{-2}

$$v_{\max} = \sqrt{\mu r \mathrm{g}} = \sqrt{0.3 \times 300 \times 10} = 30\ \mathrm{ms}^{-1}$$

$$= 30 \times \frac{18}{5}\ \mathrm{km\ h}^{-1} = 108\ \mathrm{km\ h}^{-1}$$