Single Correct Answer Type

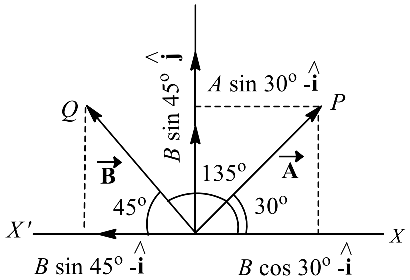
1 **(a)**

Here $\vec{A} - \overrightarrow{OP} = 10$ units along OP

$\vec{B} - (\overrightarrow{OQ}) = 10$ units along OQ

$\therefore \quad \angle XOP = 30°$ and $\angle XOQ = 135°$

$\therefore \quad \angle QOX' = 180° - 135° = 45°$



Resolving $\vec{A}$ and $\vec{B}$ into two rectangular components we have $A\cos 30°$ along OX and $A\sin 30°$ along OY. $B\cos 45°$ along OX' *and* $B\sin 45°$ along OY'.

Resultant component force along X-axis.

$(A\cos 30° - B\sin 45)\hat{i}$

$= (10 \times \sqrt{3}/2 - 10 \times 1/\sqrt{2})\hat{i} = 1.59\hat{i}$

Resultant component force along Y-axis

$= (A\sin 30° + B\sin 45°)\hat{j}$

$= (10 \times 1/2 + 10 \times 1/\sqrt{2})\hat{j} = 12.07\hat{j}$

2 **(c)**

$\vec{A} + \vec{B} = \vec{C}$ (given)

So, it is given that $\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$

$\therefore \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

$3^2 = 3 + 3 + 2 \times 3 \times \cos\theta$

$3 = 6\cos\theta$ or $\cos\theta = \frac{1}{2} \quad \Rightarrow \theta\ 60°$

3 **(a)**

$\vec{A} \cdot \vec{B} = 0 \Rightarrow \vec{A} \perp \vec{B}$

Now, $\vec{A} \times \vec{B} = \vec{1}$ or $AB\sin\theta = 1$

$AB\sin 90° = 1$ or $AB = 1 \Rightarrow A = 1$ and $B = 1$

So, $\vec{A}$ and $\vec{B}$ are perpendicular unit vectors.

4 **(c)**

Here, $v_{\max} = ?, r = 18$ m, $g = 10\ \mathrm{ms^{-2}}$

$\mu = 0.2$

$$\frac{mv_{\max}^2}{r} = F = \mu R = \mu mg$$

$v_{\max} = \sqrt{\mu rg} = \sqrt{0.2 \times 18 \times 10} = 6\ \mathrm{ms^{-1}}$

$= 6 \times \frac{18}{5}\ \mathrm{km\ h^{-1}} = 21.6\ \mathrm{km\ h^{-1}}$

5 **(b)**

$v = \sqrt{5gR}$

When $R' = \frac{R}{4}$

$v' = \sqrt{5gR'} = \sqrt{5gR/4} = \frac{1}{2}\sqrt{5gR} = \frac{1}{2}v$

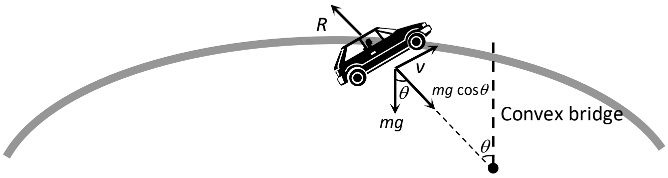
6 **(b)**

$T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$

or $T = ml\omega^2 = ml\left(2\pi \times \frac{2}{\pi}\right)^2 = 16ml$

7 **(a)**

$R = mg\cos\theta - \frac{mv^2}{r}$



When θ decreases $\cos\theta$ increases $i.e.$, R increases

8. **(a)**

Area of parallelogram $= |A \times B|$

$AB\sin\theta = \frac{1}{2}AB$

$\therefore \sin\theta = \frac{1}{2}, \theta = 30°$

9. **(a)**

In this problem it is assumed that particle although moving in a vertical loop but its speed remain constant

Tension at lowest point $T_{\max} = \frac{mv^2}{r} + mg$

Tension at highest point $T_{\min} = \frac{mv^2}{r} - mg$

$\frac{T_{\max}}{T_{\min}} = \frac{\frac{mv^2}{r} + mg}{\frac{mv^2}{r} - mg} = \frac{5}{3}$

By solving we get, $v = \sqrt{4gr} = \sqrt{4 \times 9.8 \times 2.5} = \sqrt{98}\, m/s$

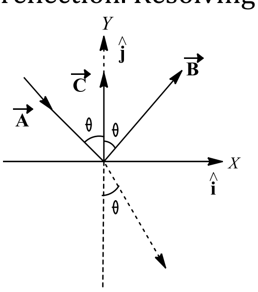
10. **(c)**

$F = m\omega^2 R \ \therefore F \propto R$ (m and ω are constant)

If radius of the path is halved, then force will also become half

11. **(d)**

Let $\vec{A}, \vec{B}$ and $\vec{C}$ be as shown in figure. Let θ be the angle of incidence, which is also equal to the angle of reflection. Resolving these vectors in rectangular components, we have



$\vec{A} = \sin\theta\hat{i} - \cos\theta\hat{j}$

$\vec{B} = \sin\theta\hat{i} + \cos\theta\hat{j}$

$\vec{B} - \vec{A} = 2\cos\theta\,\hat{j}$

or $\vec{B} = \vec{A} + 2\cos\theta\,\hat{j}$

Now $\vec{A}.\vec{C} = 2\cos\theta\hat{j}$ or $\vec{B} = \vec{A}\cos\theta\,\hat{j}$

$\therefore\ \vec{B} = \vec{A} - 2(\vec{A}.\vec{C})\hat{j}$ or $\vec{B} = \vec{A} - 2(\vec{A}.\vec{C})\vec{C}$ (as $\hat{j} = \vec{C}$)

12. **(c)**

In projectile motion given angular projection, the horizontal component velocity remains unchanged. Hence

$v\cos\alpha = u\cos\theta$ or $v = u\cos\theta\sec\alpha$

13. **(d)**

$$s = 0 \times 1 + \frac{1}{2} \times 9.8 \times 1 \times 1 = 4.9\text{ m}$$

14. **(d)**

Minimum speed at the highest point of vertical circular path $v = \sqrt{gR}$

15. **(c)**

When $\theta = 180°$, the particle will be at diametrically opposite point, where its velocity is opposite to the initial directions of motion. The change in momentum $= mv - (-mv) = 2mv$ (maximum). When $\theta = 360°$, the particle is at the initial position with momentum m. Change in momentum $mv - mv = 0$ (minimum)

16. **(d)**

$R = 4H\cot\theta$, if $\theta = 45°$ then $R = 4H \Rightarrow \frac{R}{H} = \frac{4}{1}$

17. **(b)**

Maximum tension in the thread is given by

$$T_{\max} = mg + \frac{mv^2}{r}$$

or $T_{\max} = mg + mrw^2$ $(\because v = r\omega)$

or $\omega^2 = \dfrac{T_{\max} - mg}{mr}$

Given, $T_{\max} = 37\text{ N}, \text{m} = 500\text{g} = 0.5\text{ kg}, g = \text{mg}^{-2}$,

$r = 4\text{m}$

$$\therefore\ \omega^2 = \frac{37 - 0.5 \times 10}{0.5 \times 4} = \frac{37 - 5}{2}$$

or $\omega^2 = 16$

or $\omega = 4\text{ rad s}^{-1}$

18. **(a)**

$$mg = 1 \times 10 = 10N, \frac{mv^2}{r} = \frac{1 \times (4)^2}{1} = 16$$

Tension at the top of circle $= \frac{mv^2}{r} - mg = 6N$

Tension at the bottom of circle $= \frac{mv^2}{r} + mg = 26N$

19. **(a)**

Centripetal force

$$\frac{mv^2}{R} = ma$$

or $a = \dfrac{v^2}{R}$

$$\therefore\ \frac{a_1}{a_2} = \frac{v_1^2}{v_2^2}$$

Here, $v_1 = v,\ v_2 = 2v,\quad a_1 = a$

$$\therefore\ \frac{a}{a_2} = \frac{v^2}{(2v)^2} = \frac{1}{4}$$

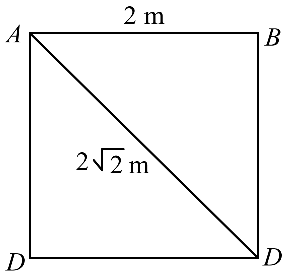
or $a_2 = 4a$

20. **(d)**

Displacement is distance from initial to final position In 40s cyclist completes =1 round

$\therefore$ In 3 min(180 s) cyclist will complete $= 4\frac{1}{2}$ round Displacement for 4 round is zero.

Displacement for $\frac{l}{2}$ round $=$ length of diagonal $= 2\sqrt{2}$m.



21. **(d)**

$B_x = 10 - 6 = 4$ and $B_y = 9 - 6 = 3$

so, $B = \left(B_x^2 + B_y^2\right)^{1/2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

$= \sqrt{25} = 5$

22. **(c)**

$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$

At $t = 2\ s, \omega = 6 \times (2)^2 = 24\ rad/s$

23. **(a)**

Here, $r = 92$ m, $v = 26$ ms^{-1}, $\mu =$?

As $\frac{mv^2}{r} = F = \mu R = \mu\, mg$

$\mu = \frac{v^2}{rg} = \frac{26 \times 26}{92 \times 9.8} = 0.75$

24. **(c)**

$v = \sqrt{\mu\, rg} = \sqrt{0.25 \times 80 \times 9.8} = 14$ ms^{-1}

25. **(a)**

$\left|\overrightarrow{\Delta v}\right| = 2v \sin(\theta/2) = 2v \sin\left(\frac{90}{2}\right) = 2v \sin 45 = v\sqrt{2}$

26. **(b)**

Let $\hat{A} + \hat{B} = \hat{R}$ then using parallelogram law of vectors we have

$1 = (1^2 + 1^2 + 2.1.1\cos\theta)^{1/2}$

or $\quad 1 = 2(1 + \cos\theta)$

or $\quad \frac{1}{2} - 1 = \cos\theta$

or $\quad \cos\theta = -\frac{1}{2} = \cos 120°$

or $\quad \theta = 120°$

$\therefore\ |\hat{A} - \hat{B}| = |\hat{A} + (-\hat{B})|$.

Now the angle between $\hat{A}$ and $-\hat{B}$ is 60°

The resultant of $|\hat{A} + (-\hat{B})|$

$(1^2 + 1^2 + 2 \times 1 \times 1 \times \cos 60°)^{1/2}$

$= \sqrt{3}$

27. **(d)**

We know that if two stones have same horizontal range, then this implies that both are projected at θ

and $90° - \theta$.

Given, $\theta = \frac{\pi}{3} = 60°$

$\therefore \; 90° - \theta = 90° - 60° = 30°$

For first stone,

Maximum height $= 102 = \frac{u^2 \sin^2 60°}{2g}$

For second stone,

Maximum height, $h = \frac{u^2 \sin^2 30°}{2g}$

$\therefore \; \frac{h}{102} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2}$

or $h = 102 \times \frac{1/4}{3/4} = 34$ m

28. **(b)**

$\vec{L} = \vec{r} \times m\vec{v} = H\, mv\cos\theta = \frac{v \sin^2\theta}{2g} mv\cos\theta = \frac{mv^3}{4\sqrt{2}g}$

29. **(c)**

$\frac{v^2}{g} = 100$ or $v^2 = 100$ g

$h_{\text{max.}} = \frac{v^2}{2g} = \frac{100g}{2g} = 50$m

30. **(b)**

$\omega_1 = 2\pi r = 2\pi \times 20, \omega_2 = 0, t = 20a, \alpha = ?$

As $\omega_2 = \omega_1 + \alpha t$

$\therefore \; \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{40\pi - 0}{20} = 2\pi$ rad s^{-2}

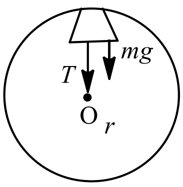
31. **(d)**

Maximum range $= \frac{u^2}{g} = 100\, m$

Maximum height $= \frac{u^2}{2g} = \frac{100}{2} = 50\, m$

32. **(b)**

When a body is revolving in circular motion it is acted upon by a centripetal force directed towards the center. Water will not fall if weight is balanced by centripetal force. Therefore



$mg = \frac{mv^2}{r}$

$\Rightarrow \; v^2 = rg \quad \dots \text{(i)}$

Circumference of a circle is $2\pi r$.

Time of revoluation $= \frac{2\pi r}{v}$

Putting the value of v from Eq. (i), we get

$T = \frac{2\pi r}{\sqrt{gr}} = 2\pi\sqrt{\frac{r}{g}}$

Given, $r = 4$ m, $g = 9.8\ \frac{\text{m}^2}{\text{s}}$

$\therefore T = 2\pi\sqrt{\frac{4}{9.8}}$

$\Longrightarrow T = \frac{4\pi}{\sqrt{9.8}} = 4\text{s}$

33. **(b)**

Centripetal force = breaking force

$\Rightarrow m\omega^2 r =$ breaking stress $\times$ cross sectional area

$\Rightarrow m\omega^2 r = p \times A \Rightarrow \omega = \sqrt{\frac{p \times A}{mr}} = \sqrt{\frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3}}$

$\therefore\ \omega = 4\ rad/sec$

34. **(d)**

$\theta = 2\pi n = \omega_0 t + \frac{1}{2}\alpha t^2$

$2\pi \times 10 = \frac{1}{2}\ \alpha 4^2$ or $\alpha = \frac{40\pi}{16}$

Let it make N rotations in the first 8 s

Then, $2\pi N = \frac{1}{2}\alpha\ 8^2$

or $N = \frac{1}{2\pi} \times \frac{64}{2} \times \frac{40\pi}{16} = 40$

$\therefore$ The required number of rotations

$= 40 - 10 = 30$

35. **(d)**

$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgL$

$\Rightarrow v = \sqrt{u^2 - 2gL}$

$|\vec{v} - \vec{u}| = \sqrt{u^2 + v^2} = \sqrt{u^2 + u^2 - 2gL} = \sqrt{2(u^2 - gL)}$

36. **(a)**

Given, $\omega_1 = 2\pi \times 400$ rad s^{-1}

$\omega_2 = 2\pi \times 200$ rad s^{-1}

$\therefore \alpha = \frac{2\pi(400-200)}{2} = 200\pi$ rad s^{-2}

37. **(c)**

In uniform circular motion tangential acceleration remains zero but magnitude of radial acceleration remains constant.

38. **(b)**

Horizontal range

$R = \frac{u^2 \sin 2\theta}{g}$ ...(i)

Maximum height

$H = \frac{u^2 \sin^2 \theta}{2g}$ ...(ii)

Here (i)=(ii)

$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

$2\cos\theta = \frac{\sin\theta}{2}$

$\theta = \tan^{-1}(4)$

39. **(d)**

For critical condition at the highest point $\omega = \sqrt{g/R}$

$\Rightarrow T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{R/g} = 2 \times 3.14\sqrt{4/9.8} = 4$ sec

40. **(d)**

$\tan\theta = \dfrac{L}{A}$

$\tan 30° = \dfrac{10v}{3400}$

$v = \dfrac{340}{\sqrt{3}} = 196.3$ m/s

41. **(d)**

$v_{\max} = \sqrt{\mu r g} = \sqrt{0.2 \times 100 \times 9.8} = 14 m/s$

42. **(b)**

$F = \frac{mv^2}{r}$. For same mass and same speed if radius is doubled then force should be halved

43. **(a)**

$R = \dfrac{v^2 \sin 2\theta}{g} = 200, T = \dfrac{2v \sin\theta}{g} = 5$

Dividing, $\frac{v^2 \times 2 \sin\theta \cos\theta}{g} \times \frac{g}{2v \sin\theta} = \frac{200}{5} = 40$

or $v \cos\theta = 40\text{ms}^{-1}$

It may be noted here that the horizontal component of the velocity of projection remains the same during the flight of the projectile

44. **(a)**

$R_{\max} = R = \dfrac{u^2}{g}$

$\Longrightarrow u^2 = Rg$

Now, as range $= \dfrac{u^2 \sin 2\theta}{g}$

then $\dfrac{R}{2} = \dfrac{Rg \sin 2\theta}{g}$

$\Longrightarrow \sin 2\theta = \dfrac{1}{2} = \sin 30° \Longrightarrow \theta = 15°$

45. **(c)**

Velocity at the lowest point, $v = \sqrt{2gl}$

At the lowest point, the tension in the string

$T = mg + \dfrac{mv^2}{l} = mg + \dfrac{m}{l}(2gl) = 3mg$

46. **(d)**

Tension is string = centrifugal force

In first case, $F = m\, r\, \omega^2$

In second case, $F' = m(2r)(2\omega)^2 = 8mr\,\omega^2 = 8\,F$

47. **(d)**

At 45°, $v_x = v_y$

or $u_x = u_y - gt$

$\therefore \ t = \dfrac{u_y - u_x}{g}$

$= \dfrac{40(\sin 60° - \sin 30°)}{9.8} = 1.5$ s

48. **(c)**

$F = \frac{mv^2}{r} = \frac{500 \times (10)^2}{50} = 1000\text{N}$

49. **(a)**

At the highest point of trajectory, the velocity becomes horizontal. So, it is perpendicular to acceleration (which is directed vertically downwards)

50. **(b)**

Area in which bullet will spread $= \pi r^2$

For maximum area, $r = R_{\text{max}} = \frac{v^2}{g}$ [When $\theta = 45°$]

Maximum area $\pi R_{\text{max}}^2 = \pi \left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$