

250 (c)

$$\vec{A} \cdot \vec{B} = AB \cos \theta = 6$$

$$\text{and } |\vec{A} \times \vec{B}| = AB \sin \theta = 6\sqrt{3}$$

$$\therefore \frac{AB \sin \theta}{AB \cos \theta} = \frac{6\sqrt{3}}{6} = \sqrt{3}$$

$$\text{or } \tan \theta = \sqrt{3}$$

$$\text{and } \theta = 60^\circ$$

251 (c)

$$\text{Using relation } \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta_1 = \frac{1}{2} (\alpha) (2)^2 = 2\alpha \quad \dots(i)$$

$$\text{As } \omega_0 = 0, t = 2 \text{ sec}$$

$$\text{Now using same equation for } t = 4 \text{ sec}, \omega_0 = 0$$

$$\theta_1 + \theta_2 = \frac{1}{2} \alpha (4)^2 = 8\alpha \quad \dots(ii)$$

$$\text{From (i) and (ii), } \theta_1 = 2\alpha \text{ and } \theta_2 = 6\alpha \therefore \frac{\theta_2}{\theta_1} = 3$$

252 (b)

$$\vec{S} = (10\hat{i} - 2\hat{j} + 7\hat{k}) - (6\hat{i} + 5\hat{j} - 3\hat{k}) = 4\hat{i} - 7\hat{j} + 10\hat{k}$$

$$\vec{W} = \vec{F} \cdot \vec{S}$$

$$= (10\hat{i} - 3\hat{j} + 6\hat{k}) \cdot (4\hat{i} - 7\hat{j} + 10\hat{k})$$

$$= (40 + 21 + 60) \text{ J} = 121 \text{ J}$$

253 (b)

The amplitude is the radius of the circle

$$R = \frac{0.8}{2} = 0.4 \text{ m}$$

The frequency of the shadow is the same as that of the circular motion, so

$$\omega = 30 \text{ rev/min}$$

$$= 0.5 \text{ rev/s} = \pi \text{ rad s}^{-1}$$

$$\text{and } v = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5 \text{ Hz.}$$

254 (c)

$$S_x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow S_x = \frac{1}{2} \times 6 \times 16 = 48 \text{ m}$$

$$S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow S_y = \frac{1}{2} \times 8 \times 16 = 64 \text{ m}$$

$$S = \sqrt{S_x^2 + S_y^2} = 80 \text{ m}$$

255 (a)

$$\text{Here, } r = 92 \text{ m}, v = 26 \text{ ms}^{-1}, \mu = ?$$

$$\text{As } \frac{mv^2}{r} = F = \mu R = \mu mg$$

$$\mu = \frac{v^2}{rg} = \frac{26 \times 26}{92 \times 9.8} = 0.75$$

256 (b)

258 (b)

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2 \text{ m/s}$$

260 (c)

Given condition $h_1 = h_2$

$$u_1^2 \sin^2 45^\circ = u_2^2 \sin^2 \theta$$

$$\sin^2 \theta = \frac{u_1^2}{u_2^2} \sin^2 45^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

262 **(b)**

$$E' = E \cos^2 \theta = E \cos^2 (45^\circ) = \frac{E}{2}$$

264 **(a)**

When particle moves in a circle, then the resultant force must act towards the centre and its magnitude

$$F \text{ must satisfy, } F = \frac{mv^2}{l}$$

This resultant force is directed towards the centre and it is called centripetal force. This force originates from the tension T

$$\text{Hence, } F = \frac{mv^2}{l} = T$$

266 **(c)**

$$\text{Displacement, } \vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i}) = a\hat{j}$$

$$\vec{F} = -K(y\hat{i} + x\hat{j}) = -K(a\hat{i} + a\hat{j})$$

$$\text{Workdone, } W = \vec{F} \cdot \vec{r}$$

$$= -K(a\hat{i} + a\hat{j}) \cdot a\hat{j} = -Ka^2$$

267 **(c)**

At the two points of the trajectory during projection, the horizontal component of the velocity is the same

$$\Rightarrow u \cos 60^\circ = v \cos 45^\circ$$

$$\Rightarrow 147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}} \Rightarrow v = \frac{147}{\sqrt{2}} \text{ m/s}$$

$$\text{Vertical component of } u = u \sin 60^\circ = \frac{147\sqrt{3}}{2} \text{ m}$$

$$\text{Vertical component of } v = v \sin 45^\circ = \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{147}{2} \text{ m}$$

$$\text{but } v_y = u_y + a_y^t \Rightarrow \frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8t$$

$$\Rightarrow 9.8t = \frac{147}{2}(\sqrt{3} - 1) \Rightarrow t = 5.49 \text{ s}$$

268 **(b)**

$$\cos \theta = \frac{(\hat{k}) \cdot (\hat{i} + \hat{j} + \sqrt{2}\hat{k})}{1\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}$$

$$\text{or } \cos \theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad \text{or } \theta = 45^\circ$$

273 **(d)**

Net displacement in one loop = 0

$$\text{Average velocity} = \frac{\text{net displacement}}{\text{time}} = \frac{0}{t} = 0$$

$$\text{Distance travelled in one rotation (loop)} = 2\pi r$$

$$\therefore \text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{t}$$

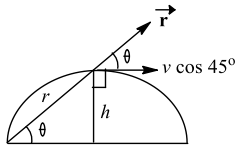
$$= \frac{2 \times 3.14 \times 100}{62.8} = 10 \text{ m/s.}$$

275 **(b)**

The angular momentum of a particle is given by

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$L = mvr \sin \theta$$



From figure,

$$L = rm(v \cos 45^\circ) \sin \theta$$

$$= \frac{mv}{\sqrt{2}} (r \sin \theta)$$

$$= \frac{mvh}{\sqrt{2}} \left(\because \sin \theta = \frac{h}{r} \right)$$

277 **(c)**

$$\text{Tension} = \text{Centrifugal force} + \text{weight} = \frac{mv^2}{r} + mg$$

278 **(a)**

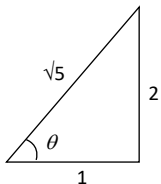
$$R = 2H \text{ Given}$$

$$\text{We know } R = 4H \cot \theta \Rightarrow \cot \theta = \frac{1}{2}$$

$$\text{From triangle we can say that } \sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = \frac{1}{\sqrt{5}}$$

$$\therefore \text{Range of projectile } R = \frac{2v^2 \sin \theta \cos \theta}{g}$$

$$= \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$



281 **(c)**

$$v = \sqrt{2gl(1 - \cos \theta)} = \sqrt{2 \times 9.8 \times 2(1 - \cos 60^\circ)} = 4.43 \text{ m/s}$$

283 **(b)**

As the speed is constant throughout the circular motion therefore its average speed is equal to instantaneous speed

287 **(b)**

For maximum range $\theta = 45^\circ$

$$\frac{R_{\max}}{T^2} = \frac{u^2 \sin 2\theta}{g} \bigg/ \frac{4u^2 \sin^2 \theta}{g^2}$$

$$\Rightarrow \frac{R_{\max}}{T^2} = \frac{\sin 90^\circ \times g}{4 \times \sin^2 45^\circ} = \frac{49}{10}$$

288 **(c)**

Taking vertical downward motion of projectile from point of projection to ground, we have

$$u = -50 \sin 30^\circ = -25 \text{ ms}^{-1}$$

$$a = +10 \text{ ms}^{-2}, s = 70 \text{ m}, t = ?$$

$$\therefore s = ut + \frac{1}{2}at^2;$$

So, $70 = -25 \times t + \frac{1}{2} \times 10 \times t^2$
 or $5t^2 - 25t - 70 = 0$ or $t^2 - 5t - 14 = 0$
 On solving $t = 7$ s

289 **(a)**

Using $v^2 - u^2 = 2as$, we get

$$s = \frac{v^2}{2g}$$

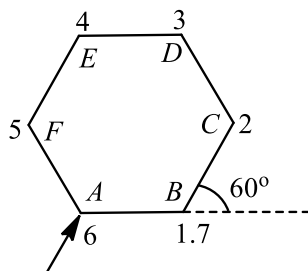
$$\text{Now, } \frac{v^2 \sin 2\theta}{g} = \frac{v^2}{2g} \text{ or } \sin 2\theta = \frac{1}{2}$$

$$\text{or } \sin 2\theta = \sin 30^\circ \text{ or } \theta = 15^\circ$$

The other possible angle of projection is $(90^\circ - 15^\circ)$,
 ie, 75°

291 **(a)**

In 6 turns each of 60° , the cyclist traversed a regular hexagon path having each side 100 m. So, at 7th turn, he will be again at



Starting point

Point B (as shown) which is at distance 100 m from starting point A. Hence, net displacement of cyclist is 100 m.

292 **(a)**

When speed is constant in circular motion, it means work done by centripetal force is zero

294 **(b)**

$$\begin{aligned} \text{Change in momentum} &= 2mu \sin \theta \\ &= 2 \times 0.5 \times 98 \times \sin 30 = 49 \text{ N}\cdot\text{s} \end{aligned}$$

295 **(d)**

$$\text{At highest point } \frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$$

296 **(b)**

$$\begin{aligned} \text{Difference in KE} &= \frac{1}{2}m[(\sqrt{5gr})^2 - \sqrt{gr}]^2 \\ &= 2mgr = 2 \times 1 \times 10 \times 1 = 20 \text{ J} \end{aligned}$$

300 **(b)**

$$v = 72 \text{ km/hour} = 20 \text{ m/sec}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{20 \times 20}\right) = \tan^{-1}(2)$$

302 **(c)**

Equation of trajectory for oblique projectile motion

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Substituting $x = D$ and $u = v_0$

$$h = D \tan \theta - \frac{gD^2}{2u_0^2 \cos^2 \theta}$$

303 **(d)**

Given $\theta_1 = \pi/3 = 30^\circ$

Horizontal range is same if $\theta_1 + \theta_2 = 90^\circ$

$$\therefore \theta_2 = 90^\circ - 30^\circ = 60^\circ$$

$$y_1 = \frac{u^2 \sin^2 30^\circ}{2g} \text{ and } y_2 = \frac{u^2 \sin^2 60^\circ}{2g}$$

$$\therefore \frac{y_2}{y_1} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \left(\frac{1/4}{\sqrt{3}/4} \right)^2 = \frac{1}{2} \text{ or } y_2 = \frac{y_1}{2}$$

306 (a)

$$\text{Given } (\hat{i} + 2\hat{j} - \hat{k}) + (\hat{i} - \hat{j} + 2\hat{k}) + \vec{C} = \hat{j}$$

$$\therefore \vec{C} = \hat{j} - (\hat{i} - 2\hat{j} - \hat{k}) - (\hat{i} + \hat{j} + 2\hat{k})$$

$$= -2\hat{i} - \hat{k}$$

307 (b)

For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the center of circle or is always along radius of the circle. Since, tangential vector is perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

308 (a)

$$\frac{u^2 \sin 2\theta}{g} = 4\sqrt{3} \times \frac{u^2 \sin \theta}{2g}$$

$$\text{or } \frac{u^2}{g} 2 \sin \theta \cos \theta = 2\sqrt{3} \frac{u^2}{g} \sin^2 \theta$$

309 (b)

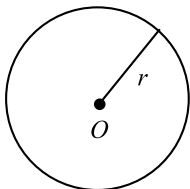
The time taken by the particle for one complete revolution.

$$t = \frac{2\pi r}{\text{speed}}$$

$$= \frac{2 \times 3.14 \times 100}{31.4} = 20\text{s}$$

Hence, average speed is

$$v_{\text{av}} = \frac{2 \times 3.14 \times 100}{20} = 31.4 \text{ ms}^{-1}$$



311 (d)

At the two point of the trajectory during projectile motion, the horizontal component of the velocity is same. Then,

$$u \cos 60^\circ = v \cos 45^\circ$$

$$147 \times \frac{1}{2} = v \times \frac{1}{\sqrt{2}} \text{ or } v = \frac{147 \text{ m}}{\sqrt{2} \text{ s}}$$

$$\text{Initially, } u_y = u \sin 60^\circ = \frac{147\sqrt{3}}{2} \text{ m/s}$$

$$\text{Finally, } v_y = v \sin 45^\circ = \frac{147}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{147}{2} \text{ m/s}$$

$$\text{But } v_y = u_y + a_y t \text{ or } \frac{147}{2} = \frac{147\sqrt{3}}{2} - 9.8 t$$

$$9.8 t = \frac{147}{2} (\sqrt{3} - 1) \text{ or } t = 5.49 \text{ s}$$

312 (c)

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 6t) = 6t - 6. \text{ At } t = 1, v_x = 0$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(t^2 - 2t) = 2t - 2. \text{ At } t = 1, v_y = 0$$

$$\text{Hence } v = \sqrt{v_x^2 + v_y^2} = 0$$

314 (a)

For horizontal planes potential energy remains constant equal to zero, if we assume surface to be the zero level.

318 (b)

$$F = mr\omega^2 = mr(2\pi v)^2 \text{ i.e., } F \propto v^2$$

$$\frac{2F}{F} = \left(\frac{v'}{v}\right)^2 \text{ or } v' = v\sqrt{2} = 5\sqrt{2} = 7\text{rpm}$$

319 (b)

Since $\vec{F} = 4\hat{i} - 3\hat{j}$ is lying in $X - Y$ plane, hence the vector perpendicular to \vec{F} must be lying perpendicular to $X - Y$ plane i.e., along Z -axis.

320 (c)

$$\mu = \frac{v^2}{rg} = \frac{(60 \times 5/18)^2}{40 \times 9.8} = 0.71$$

321 (c)

$$F = m\omega^2 R \therefore R \propto \frac{1}{\omega^2} \text{ (} m \text{ and } F \text{ are constant)}$$

If ω is doubled then radius will become $1/4$ times i.e. $R/4$

324 (c)

$$h = v \sin \theta t - \frac{1}{2}gt^2$$

$$\text{or } \frac{1}{2}gt^2 - v \sin \theta t + h = 0$$

$$t_1 + t_2 = -\frac{-v \sin \theta}{\frac{1}{2}g} \text{ or } t_1 + t_2 = \frac{2v \sin \theta}{g} = T$$

$$\text{or } T = (1 + 3)s = 4s$$

325 (c)

$$V_{\max} = \sqrt{\mu rg} = \sqrt{0.75 \times 60 \times 9.8} = 21\text{m/s}$$

326 (a)

There is no change in the angular velocity, when speed is constant

328 (c)

Let t be time taken by the bullet to hit the target

$$\therefore 200\text{ m} = 2000\text{ ms}^{-1}t$$

$$\Rightarrow t = \frac{200\text{m}}{2000\text{ms}^{-1}} = \frac{1}{10}\text{s}$$

For vertical motion,

$$\text{Here } u = 0$$

$$\therefore h = \frac{1}{2}gt^2$$

$$h = \frac{1}{2} \times 10 \times \left(\frac{1}{10}\right)^2 = \frac{1}{20}\text{m} = 5\text{ cm}$$

\therefore Gun should be aimed 5 cm above the target

329 (b)

Component of velocity perpendicular to plane remains the same (in opposite direction)

$$\text{i.e., } u \sin \theta = 20 \sin 30^\circ = 10\text{ ms}^{-1}$$

330 (c)

$$\text{Total time of flight} = \frac{2u \sin \theta}{g} = \frac{2 \times 50 \times 1}{2 \times 10} = 5s$$

Time to cross the wall = 3 sec (Given)

Time in air after crossing the wall = $(5 - 3) = 2 \text{ sec}$

\therefore Distance travelled beyond the wall = $(u \cos \theta)t$

$$= 50 \times \frac{\sqrt{3}}{2} \times 2 = 86.6 \text{ m}$$

333 (a)

$$T = m\omega^2 r \Rightarrow \omega \propto \sqrt{T} \therefore \frac{\omega_2}{\omega_1} = \sqrt{\frac{1}{4}} \Rightarrow \omega_2 = \frac{\omega_1}{2} = 5 \text{ rpm}$$